Advanced Graph Algorithms and Optimization Spring 2021

The Cut-Matching Game: Expanders via Max Flow

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The exercises for this week will not count toward your grade, but you are highly encouraged to solve them all.

Exercise 1: Proof that $\mathbf{\Pi}^t$ is doubly-stochastic

In chapter 14, we stated that the matrix $\mathbf{\Pi}^t$ is doubly-stochastic. We now ask you to prove it.

Exercise 2: Run-time Analysis of the Cut-Matching Algorithm

Analyze the run-time of the Cut-Matching algorithm to establish a total run-time of $O(\log^2 n) \cdot T_{max-flow}(G) + \tilde{O}(m)$[1]. You may assume that computing a random vector $\boldsymbol{r}$ takes $O(n)$ time, and you may use without proof that using link-cut trees you can find a path decomposition in $\tilde{O}(m)$ time.

Exercise 3: Bounding the Diameter of an Expander

Let G be a connected, unweighted graph that is a ϕ-expander with regard to conductance. Prove that the diameter of G is $O(\log m/\phi)$. You may find it useful that $\phi \leq 1$ and that $e^x < 1 + x + x^2$ for $x < 1.79$.

Hint: Fix any pair of vertices s, t in G. Then think about running BFS explorations simultaneously from s and t.

[1] Technically, we run the max flow procedure on graphs G with two additional vertices and n additional edges. You should ignore this subtlety as it will not impact the asymptotics for any max-flow algorithm known.