ETH
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Algorithms, Probability, and Computing **Exercises KW45** **HS23**

General rules for solving exercises

- When handing in your solutions, please write your exercise group on the front sheet:

  Group A: Wed 14–16 CAB G 56

  Group B: Wed 14–16 CAB G 57

  Group C: Wed 16–18 CAB G 56

  Group D: Wed 16–18 CAB G 57

- This is a theory course, which means: if an exercise does not explicitly say "you do not need to prove your answer", then a formal proof is **always** required.

The following exercises will be discussed in the exercise class on November 8, 2023. These are "**in-class**" exercises, which means that we do not expect you to solve them before the exercise session. Instead, your teaching assistant will solve them with you in class.

Exercise 1

Show that the dual of

$$\begin{array}{lrcl} \text{maximize} & c^{\mathsf{T}}x & & \\ \text{subject to} & Ax & \leq & b, \end{array} \tag{1}$$

is

$$\begin{array}{lrcl} \text{minimize} & b^{\mathsf{T}}y & & \\ \text{subject to} & A^{\mathsf{T}}y & = & c \\ & y & \geq & 0. \end{array} \tag{2}$$

Exercise 2

Find an example of a specific linear program (P) for each of the cases in Theorem 4.6.

Exercise 3

Work out a different way of using an algorithm for the feasibility problem for solving linear programs, based on the idea of binary search for the optimum value. Theorem 4.2 is useful for analyzing the number of steps of the binary search.

Exercise 4

Let $\mathbf{v} \in \mathbb{R}^n$ be a unit vector. Find a rotation R with $R(\mathbf{e}_1) = \mathbf{v}$ (write down the matrix of R w.r.t. the standard basis of $\mathbb{R}^n$).