

General rules for solving exercises

- When handing in your solutions, please write your exercise group on the front sheet:

Group A: Wed 14–16 CAB G 56

Group B: Wed 14–16 CAB G 57

Group C: Wed 16–18 CAB G 56

Group D: Wed 16–18 CAB G 57

- This is a theory course, which means: if an exercise does not explicitly say “you do not need to prove your answer”, then a formal proof is **always** required.

The following exercises will be discussed in the exercise classes on November 13, 2024. Please hand in your solutions via Moodle, no later than 2 pm at November 12.

Exercise 1

Suppose that we have an oracle that, given a system of linear inequalities, decides its feasibility (outputs YES or NO). Design an algorithm that computes a solution of a given system of linear equations and inequalities, provided that one exists, in polynomial time and with polynomially many calls of the oracle.

- (a) How can we proceed if there are only equations in the system?
- (b) If there is at least one inequality, use the oracle to check if there is a solution satisfying that inequality with equality, and take appropriate actions depending on the outcome.

Exercise 2

Recall the definition of the Subtour LP:

$$\begin{aligned} \text{minimize } c^T x \text{ subject to } & \sum_{e \in \delta(v)} x_e = 2 \text{ for all } v \in V \\ & \sum_{e \in \delta(S)} x_e \geq 2 \text{ for all } S \subseteq V \text{ with } \emptyset \neq S \neq V \\ & 1 \geq x_e \geq 0 \text{ for all } e \in E. \end{aligned}$$

- (i) Show that a graph $G = (V, E)$ is connected if and only if $\delta(S) \neq \emptyset$ for all $S \subseteq V$, $\emptyset \neq S \neq V$. Prove this using the most basic definition of connectedness: A graph is connected if for any two vertices $v, w \in V$ there is a path from v to w .
- (ii) Give a graph G where the subtour LP is feasible but there is no feasible integer solution.
- (iii) Assume $|V| \geq 3$. Show that the constraints " $1 \geq x_e$ " are redundant in the Subtour LP, i. e. every point $x \in \mathbf{R}^E$ that is feasible w.r.t. all other constraints also satisfies $1 \geq x_e$ for all $e \in E$.

Exercise 3

Let $m \in \mathbf{N}$, $c \in \mathbf{R}^m$, $S \subseteq \mathbf{R}^m$ finite and $P := \text{conv}(S)$. Show that we have

$$\min_{x \in P} c^T x = \min_{x \in S} c^T x.$$